JEE 2002 - SOLUTIONS - MATHEMATICS

Solution1: a, A_1, A_2, b are in arithmetic progression

$\Rightarrow$ A_1, A_2 are two arithmetic means of a, b

$$\Rightarrow A_1 = a + \frac{b-a}{3} = \frac{2a+b}{3} \qquad \text{(I)}$$

$$A_2 = a + \frac{2(b-a)}{3} = \frac{a+2b}{3} \qquad \text{(II)}$$

a, G_1, G_2, b are in geometric progression

Let r be the common ratio.

$G_1 = ar,\ G_2 = ar^2,\ b = ar^3$

$\Rightarrow r = (b/a)^{1/3}$ (III)

$$\Rightarrow G_1 = a\left(\frac{b}{a}\right)^{1/3} = b^{1/3}\,a^{2/3} \qquad \text{(IV)}$$

a, H_1, H_2, b are in the harmonic progression

$$\Rightarrow \frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{b} \text{ are in AP}$$

Let d' be the common difference.

$$\Rightarrow \frac{1}{H_1} = \frac{1}{a} + d',\ \frac{1}{H_2} = \frac{1}{a} + 2d',\ \frac{1}{b} = \frac{1}{a} + 3d'$$

$$\Rightarrow d' = \frac{a-b}{3ab}$$

$$\Rightarrow \frac{1}{H_1} = \frac{1}{a} + \frac{a-b}{3ab} = \frac{3b+a-b}{3ab} = \frac{a+2b}{3ab}$$

$$H_1 = \frac{3ab}{a+2b} \qquad \text{(V)}$$

$$\frac{1}{H_2} = \frac{1}{a} + \frac{2(a-b)}{3ab} = \frac{3b+2a-2b}{3ab} = \frac{2a+b}{3ab}$$

$$H_2 = \frac{3ab}{2a+b} \qquad \text{(VI)}$$

$$\therefore \frac{G_1G_2}{H_1H_2} = \frac{(b^{1/3}a^{2/3})(a^{1/3}b^{2/3})}{\left(\frac{3ab}{a+2b}\right)\left(\frac{3ab}{2a+b}\right)}$$

$$= \frac{ab}{9a^2b^2}(a+2b)(2a+b)$$

$$= \frac{(a+2b)(2a+b)}{9ab}$$

$$\frac{A_1+A_2}{H_1+H_2} = \frac{\left(\frac{2a+b}{3}+\frac{a+2b}{3}\right)}{\frac{3ab}{a+2b}+\frac{3ab}{2a+b}}$$

$$= \frac{\frac{3(a+b)}{3}}{3ab\left(\frac{3(a+b)}{(a+2b)(2a+b)}\right)}$$

$$= \frac{(a+2b)(2a+b)}{9ab}$$

$$\Rightarrow \frac{G_1G_2}{H_1H_2} = \frac{A_1+A_2}{H_1+H_2} = \frac{(2a+b)(a+2b)}{9ab}$$

Solution 2:
$P(n)$: $(25)^{n+1} - 24n + 5735$ is divisible by $(24)^2$
LHS of $P(1)$: $(25)^2 - 24 + 5735$
$= (625 + 5735) - 24$
$= 6360 - 24$
$= 24(265 - 1)$
$= 24 \times 264$
$= 24 \times 24 \times 11$ is divisible by $(24)^2$
Hence, $P(1)$ is true
Let us assume that $P(k)$ is true
$\Rightarrow (25)^{k+1} - 24k + 5735$ divisible by $(24)^2$
Now, we have to prove that $P(k + 1)$ is true.
i.e. $(25)^{k+2} - 24(k + 1) + 5735$ is divisible by $(24)^2$ if $P(k)$ is true.
$(25)^{k+2} - 24(k + 1) + 5735$
$= (25^{k+1}) . 25 + 25(-24k + 5735) - 25(5735 - 24k) - 24(k + 1) + 5735$
$= 25[P(k)] - 24(5735) + 24 \times 25k - 24k - 24$
$= 25P(k) - 24[5735 - 24k + 1]$
$= 25P(k) - 24[5736 - 24k]$
$= 25P(k) - (24)^2[239 - k]$
$\Rightarrow P(k + 1)$ is true.
Hence, proved

Solution 3: Let $E = \cos \tan^{-1} \sin \cot^{-1} x$
Let $\cot^{-1} x = \theta$
$\therefore \quad x = \cot\theta \qquad (1)$

$\Rightarrow E = \cos\tan^{-1}\sin\theta$

$x = \cot\theta$

$$\Rightarrow \sin\theta = \frac{1}{\sqrt{1+\cot^2\theta}} = \frac{1}{\sqrt{1+x^2}} \qquad (2)$$

$$\Rightarrow E = \cos\tan^{-1}(\sin\theta)$$

$$= \cos\tan^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) \qquad (3)$$

$$\text{Let} \tan^{-1}\frac{1}{\sqrt{1+x^2}} = y$$

$$\frac{1}{\sqrt{1+x^2}} = \tan y \qquad (4)$$

To evaluate $E = \cos y$:

We have $\cos\theta = \frac{1}{\sqrt{1+\tan^2\theta}}$

$$\Rightarrow \cos y = \frac{1}{\sqrt{1+\tan^2 y}}$$

$$= \frac{1}{\sqrt{1+\left(\frac{1}{\sqrt{1+x^2}}\right)^2}} \text{ (from equation (4))}$$

$$= \frac{1}{\sqrt{1+\frac{1}{1+x^2}}}$$

$$= \frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}$$

$$\Rightarrow E = \frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}$$

Hence proved.

Solution 4.

Total coins = N
Number of fair coins = m
Therefore, number of biased coins = $N - m$

Case I:
Let coin drawn be fair:
Let us calculate the probability $P(A)$ of getting a head first and then a tail.
$P(A) = p(H)\, p(T)$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4} \qquad \left[\begin{array}{l} p(H) = \text{probability of getting head from fair coin} = \frac{1}{2} \\ p(T) = \text{probability of getting tail from fair coin} = \frac{1}{2} \end{array}\right]$$

Case II:
Let the coin drawn be biased:
Let us calculate the probability $P(B)$ of getting a head first and then a tail.

$$P(B) = p'(H)p'(T)$$

$$= \left(\frac{2}{3}\right)\left(\frac{1}{3}\right) = \frac{2}{9}$$

[$p'(H)$ = probability of getting a head from the biased coin.
$p'(T)$ = probability of getting a tail from the biased coin
$p'(H) = 2/3$ (given)
$p'(T) = 1-p'(H) = 1-2/3 = 1/3$]

Let us define

$$P'(A) = P(A) \times \text{probability of drawing a fair coin}$$

$$= \left(\tfrac{1}{4}\right)\left(\frac{m}{n}\right) \qquad \text{(ii)}$$

and $P'(B) = P(B) \times$ probability of drawing a biased coin.

$$= \frac{2}{9}\left(\frac{N-m}{N}\right) \qquad \text{(iii)}$$

Then from Bayes Theorem, we get,

$$\text{Probability (drawing a fair coin)} = \frac{p'(A)}{p'(A)+p'(B)} \qquad \text{(i)}$$

From equation (i), (ii), (iii) probability (of drawing a fair coin)

$$= \frac{\frac{1}{4}\frac{m}{n}}{\frac{1}{4}\frac{m}{N}+\frac{2}{9}\frac{N-m}{N}}$$

$$= \frac{\frac{m}{4}}{\frac{m}{4}+\frac{2}{9}(N-m)}$$

$$= \frac{9m}{9m+8(N-m)}$$

$$= \frac{9m}{8N+m}$$

Solution 5:

$$Z^{p+q} - Z^{p} - Z^{q} + 1 = 0$$

$$\Rightarrow (Z^{p} - 1)(Z^{q} - 1) = 0$$

$\therefore$ Either α is a p^{th} root of unity or q^{th} root of unity.

Using the properties of n^{th} root of unity:

either $1+\alpha+\alpha^{2}+ \ . \ . \ . + \alpha^{p-1} = 0$

or $1+\alpha+\alpha^{2}+ \ . \ . \ . + \alpha^{q-1} = 0$

Suppose both the equations hold simultaneously. Without loss of generalisation let $p > q$.

$$\therefore \ 1+\alpha+\alpha^{2}+ \ . \ . \ . + \alpha^{p-1} = 0$$

$$\Rightarrow 1+\alpha+\alpha^{2}+ . \ . \ . + \alpha^{q-1}+\alpha^{q}+\alpha^{q+1}+ . \ . \ . + \alpha^{p-1} = 0$$

$$\Rightarrow 0+\alpha^{q}+\alpha^{q+1}+ . \ . \ . + \alpha^{p-1} = 0$$

$$\Rightarrow \alpha^{q}\left[1+\alpha+\ldots+\alpha^{p-q-1}\right]=0$$

Now, $\alpha^{q}=1$

$\therefore$ the equation implies that

$1+\alpha+\ldots+\alpha^{p-q-1}=0$

Hence α should be the $(p - q)^{\text{th}}$ root of unity i.e., $\alpha^{p-1}=1$

$\Rightarrow$ $p - q$ is a multiple of q ($\because$ q is prime)

i.e., $p - q = nq$

$\Rightarrow$ $p = (n + 1)q$

$\Rightarrow$ p is not prime which is a contradiction.

Hence proved.

Solution 6:

Let the equation of L be:

$y = mx$ (i) ($\because$ it passes through the origin)

Let us find the point of intersection of (i) and $x + y = 1$.

Substituting $y = mx$ in $x + y = 1$,

we get $x=\dfrac{1}{m+1}$

and $y=\dfrac{m}{m+1}$

Hence the coordinates of P are $\left(\dfrac{1}{m+1},\dfrac{m}{m+1}\right)$

Similarly let us find the point of intersection of (i) with $x + y = 3$.

Substituting $y = mx$ in $x + y = 3$ we get

$$x=\frac{3}{m+1}$$

$$y=\frac{3m}{m+1}$$

Hence, the coordinates of Q are $\left(\dfrac{3}{m+1},\dfrac{3m}{m+1}\right)$

Slope of $L_1 = 2$,

since it is parallel to $2x - y = 5$.

Slope of $L_2 = -3$,

since it is parallel to $3x + y = 5$.

$\therefore$ Equation of L_1: $\left(y-\dfrac{m}{m+1}\right)=2\left(x-\dfrac{1}{m+1}\right)$ (i)

$\therefore$ Equation of L_2: $\left(y-\dfrac{3m}{m+1}\right)=-3\left(x-\dfrac{3}{m+1}\right)$ (ii)

Subtracting (ii) from (i), we get

$$\frac{2m}{m+1}=5x-\frac{11}{m+1}$$

$$\Rightarrow x=\frac{11+2m}{5(m+1)}.$$

$\Rightarrow$ $5mx + 5x = 11 + 2m$

$\Rightarrow$ $m(5x - 2) = 11 - 5x$

$\Rightarrow m = \dfrac{11 - 5x}{5x - 2}$ (iii)

Substituting this in (i) to eliminate m we get

$$y = 2x + \frac{15 - 15x}{9}$$

$\Rightarrow$ $3y = x + 5$

which is the equation of a straight line.

Hence proved.

Solution 7:

Let the equation of the straight line be:

$(y - 2) = m(x - 8)$

Substituting $x = 0$, we get, $x = \dfrac{(8m-2)}{m}$

$y = 2 - 8m$

Therefore, Q ≡ $(0, 2 - 8m)$

Substituting $y = 0$, we get,

Therefore, $P \equiv \left(\dfrac{8m-2}{m}, 0\right)$

$OP = \dfrac{8m-2}{m}$

$OQ = 2 - 8m$

$L = OP + OQ$

$$= \frac{8m-2}{m} + 2 - 8m$$

$$= \frac{-8m^2 + 10m - 2}{m}$$

Differentiating with respect to m and setting it equal to zero for extrema:

$$\frac{dL}{dm} = \frac{m(-16m + 10) - (-8m^2 + 10m - 2)}{m^2} = 0$$

$\Rightarrow -8m^2 + 2 = 0$

$\Rightarrow m^2 = \dfrac{1}{4}$

$\Rightarrow m = \pm\dfrac{1}{2}$

But m is given to be negative.

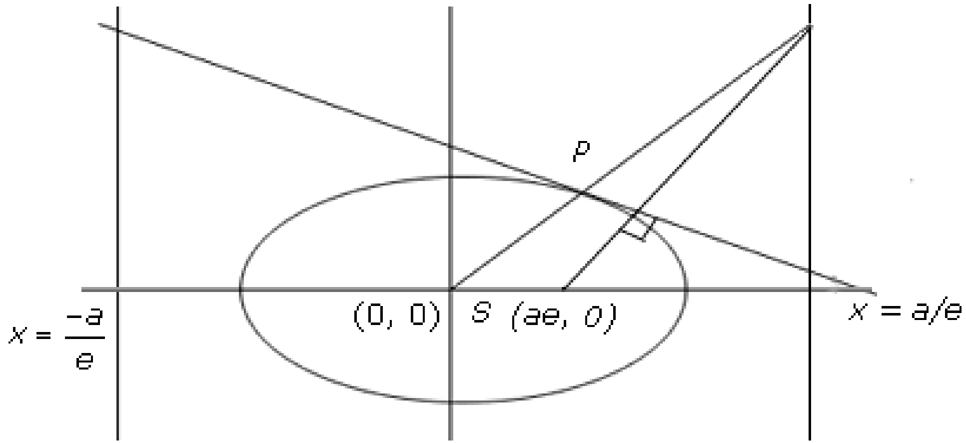
Therefore, $m = -\dfrac{1}{2}$

This m corresponds to the absolute minima (as the maxima is unbounded)

Value of absolute minima of $OP + OQ$

$$= \frac{-2 - 5 - 2}{-\frac{1}{2}} = 18$$

Solution 8:



Let the equation of ellipse be :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Let a point P on the ellipse be (a cosθ, b sinθ)
Then the equation of tangent at P is :

$$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$$

$$\Rightarrow m = \frac{-b}{a\tan\theta}$$

Equation of line L_1 joining the centre of the ellipse (0, 0) to the point P (a cosθ, b sinθ) is

$$y = \frac{b}{a}\tan\theta \, . \, x \qquad (1)$$

Slope of the line L_2 perpendicular to tangent and passing through the focus S(ae, 0) is

$$m_2 = \frac{-1}{m} = \frac{a\tan\theta}{b}$$

So equation of line L_2 is

$$y - 0 = \frac{a\tan\theta}{b}(x - ae)$$

$$\Rightarrow y = \frac{a\tan\theta}{b}(x - ae) \qquad (2)$$

Solving (1) and (2) for x, we get

$$\frac{b}{a}\tan\theta . x = \frac{a}{b}\tan\theta\,(x - ae)$$

$$\Rightarrow \frac{b^2 - a^2}{ab}x = \frac{-a^2 e}{b}$$

$$\Rightarrow \frac{a^2 - b^2}{a^2}x = ae$$

But $\frac{a^2 - b^2}{a^2} = e^2$

The equation is $e^2 x = ae$

$\Rightarrow x = a/e$ which is the equation of the corresponding directrix. Hence proved

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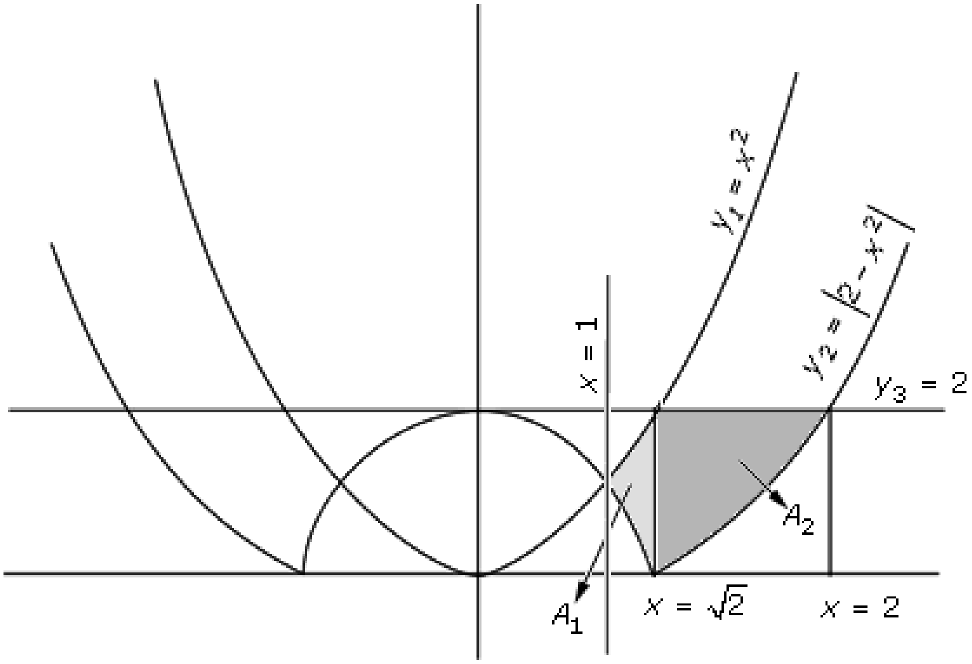
Solution 9:



Shaded area indicates the area to be calculated

$$A_1 = \int_{x=1}^{x=\sqrt{2}} \left[y_1 - y_2\right]$$

$y_2 = 2 - x^2$ for $-\sqrt{2} < x < \sqrt{2}$

So,

$$A_1 = \int_1^{\sqrt{2}} \left[x^2 - \left(2 - x^2\right)\right] dx$$

$$= \int_1^{\sqrt{2}} \left(2x^2 - 2\right) dx$$

$$= 2\int_1^{\sqrt{2}} \left(x^2 - 1\right) dx$$

$$= 2\left[\left[\frac{x^3}{3}\right]_1^{\sqrt{2}} - \left(\sqrt{2} - 1\right)\right]$$

$$= 2\left[\frac{1}{3}\left[2\sqrt{2} - 1\right] - \left[\sqrt{2} - 1\right]\right]$$

$$= \frac{4}{3} + \frac{2\sqrt{2}}{3}$$

$$A_2 = \int_{x=\sqrt{2}}^{2} [y_3 - y_2]\, dx$$

$$= \int_{\sqrt{2}}^{2} [2 - (x^2 - 2)]\, dx$$

$$= \int_{\sqrt{2}}^{2} [4 - x^2]\, dx$$

$$= 4[2 - \sqrt{2}] - \left.\frac{x^3}{3}\right|_{\sqrt{2}}^{2}$$

$$= 8 - 4\sqrt{2} - \frac{8}{3} + \frac{2\sqrt{2}}{3}$$

$$A_2 = \frac{16}{3} - \frac{10\sqrt{2}}{3}$$

$$A = A_1 + A_2$$

$$= \frac{2\sqrt{2}}{3} + \frac{4}{3} + \frac{16}{3} - \frac{10\sqrt{2}}{3}$$

$$= \frac{20}{3} - \frac{8\sqrt{2}}{3}$$

$$= \frac{20}{3} - \frac{8}{3}\sqrt{2}$$

Solution 10:

Given, $\sum_{r=1}^{3} \left(a_r + b_r + c_r\right) = 3L$

$\Rightarrow (a_1 + b_1 + c_1) + (a_2 + b_2 + c_2) + (a_3 + b_3 + c_3) = 3L$

$\Rightarrow (a_1 + a_2 + a_3) + (b_1 + b_2 + b_3) + (c_1 + c_2 + c_3) = 3L$

Now,

$$\frac{X + Y + Z}{3} \geq \left(XYZ\right)^{\frac{1}{3}}$$

AM ≥ GM

$$\Rightarrow \frac{\left(a_1 + a_2 + a_3\right) + \left(b_1 + b_2 + b_3\right) + \left(c_1 + c_2 + c_3\right)}{3} = L$$

If $X = a_1 + a_2 + a_3$

$Y = b_1 + b_2 + b_3$

$Z = c_1 + c_2 + c_3$

then, $\frac{X + Y + Z}{3} \geq \left(XYZ\right)^{\frac{1}{3}}$

$\Rightarrow L \geq (XYZ)^{1/3}$

$\Rightarrow L^3 \geq XYZ$

$\Rightarrow L^3 \geq (a_1 + a_2 + a_3)(b_1 + b_2 + b_3)(c_1 + c_2 + c_3)$

Also, $A + B + C \ge \sqrt{A^2 + B^2 + C^2}$ [since $(A + B + C)^2 - (A^2 + B^2 + C^2) = 2(AB + BC + CA) \ge 0$]

$$\Rightarrow L^3 \ge \sqrt{a_1^2 + a_2^2 + a_3^2}\sqrt{b_1^2 + b_2^2 + b_3^2}\sqrt{c_1^2 + c_2^2 + c_3^2} \qquad (1)$$

Volume of parallelopiped = $[\bar{a}\ \bar{b}\ \bar{c}]$

$= \left[\bar{a} \cdot \left(\bar{b} \times \bar{c}\right)\right] \le |\bar{a}||\bar{b}||\bar{c}|$ [equality holds for a cuboid]

$$\Rightarrow V \le \sqrt{a_1^2 + a_2^2 + a_3^2} \cdot \sqrt{b_1^2 + b_2^2 + b_3^2} \cdot \sqrt{c_1^2 + c_2^2 + c_3^2} \qquad (2)$$

From (1) and (2)

V $\le L^3$

Solution 11:

$$I = \int \left(x^{3m} + x^{2m} + x^m\right)\left(2x^{2m} + 3x^m + 6\right)^{1/m} dx, \ \ x > 0$$

Substitute $x^m = y$

Taking log,

$m \log x = \log y$

Differentiating,

$$m \frac{1}{x} dx = \frac{1}{y}\ dy$$

$$\Rightarrow dx = \frac{x}{my}\ dy$$

$$= \frac{y^{1/m}}{my}\ dy$$

$$\Rightarrow I = \int (y^3 + y^2 + y)(2y^2 + 3y + 6)^{1/m} \frac{y^{1/m}}{my}\ dy$$

$$= \frac{1}{m}\int \left[\frac{y^3 + y^2 + y}{y}\right]\left[(2y^2 + 3y + 6)^{1/m}\right] y^{\frac{1}{m}}\ dy$$

$$= \frac{1}{m}\int (y^2 + y + 1)(2y^3 + 3y^2 + 6y)^{1/m}\, dy$$

Now put $2y^3 + 3y^2 + 6y = t^m$

Differentiating both sides,

$$(6y^2 + 6y + 6)dy = mt^{m-1}\ dt$$

$$\therefore (y^2 + y + 1)\ dy = \frac{mt^{m-1}}{6}\ dt$$

$$\therefore I = \frac{1}{m}\int \frac{m}{6} t^{m-1} (t^m)^{1/m}\ dt$$

$$= \frac{1}{6}\int t^{m-1}.t \quad dt$$

$$= \frac{1}{6}\int t^{m} dt$$

$$= \frac{1}{6}\frac{t^{m+1}}{(m+1)} + c$$

$$= \frac{(2y^3 + 3y^2 + 6y)^{\frac{m+1}{m}}}{6(m+1)} + c$$

$$\therefore \; I = \frac{1}{6(m+1)}\left[2 \; x^{3m} + 3x^{2m} + 6x^{m}\right]^{\left(\frac{m+1}{m}\right)} + c$$

Solution 12:

$$f(x) = \begin{cases} x + a, \; x < 0 \\ |x - 1|, \; x \geq 0 \end{cases}$$

$$= \begin{cases} x + a, \; x < 0 \\ x - 1, \; x \geq 1 \\ 1 - x, \; 0 \leq x < 1 \end{cases}$$

$$g(x) = \begin{cases} x + 1 \qquad , \quad x < 0 \\ (x - 1)^2 + b, \text{ if } x \geq 0 \end{cases}$$

$$gof(x) = g(f(x)) = \begin{cases} f(x) + 1 \qquad , \; f(x) < 0 \\ [f(x) - 1]^2 + b, \text{ if } \; f(x) \geq 0 \end{cases}$$

Now, $f(x) < 0$

$$\Rightarrow \begin{cases} x + a < 0 \quad \text{when} \quad x < 0 \\ x - 1 < 0 \quad \text{when} \quad x \geq 1 \\ 1 - x < 0 \quad \text{when} \quad 0 \leq x < 1 \end{cases}$$

$$\Rightarrow \begin{cases} x < -a \quad \text{when} \; x < 0 \\ x < 1 \quad \text{when} \quad x \geq 1 \\ x > 1 \quad \text{when} \quad 0 \leq x < 1 \end{cases}$$

The last two cases are not possible

So, $f(x) < 0$ if $x < -a$

a is positive

$f(x) < 0$ if $x < -a$

$\Rightarrow f(x) \geq 0$ for $x > -a$

Now,

$$gof(x) = \begin{cases} f(x) + 1, \; x < -a, \text{ where } f(x) = x + a \\ [f(x) - 1]^2 + b \;, \; x \geq -a \end{cases}$$

$$gof(x) = \begin{cases} x + a + 1 & , \quad x < -a \\ (x + a - 1)^2 + b, & -a \le x < 0 \end{cases}$$

$$= (1 - x - 1)^2 + b,\ 0 \le x < 1$$

$$= x^2 + b,\ 0 \le x < 1$$

$gof(x) = (x - 1 - 1)^2 + b,\ x \ge 1$

$= (x - 2)^2 + b,\ x \ge 1$

Since, gof is continuous for all real x, therefore, $(a - 1)^2 + b = b$

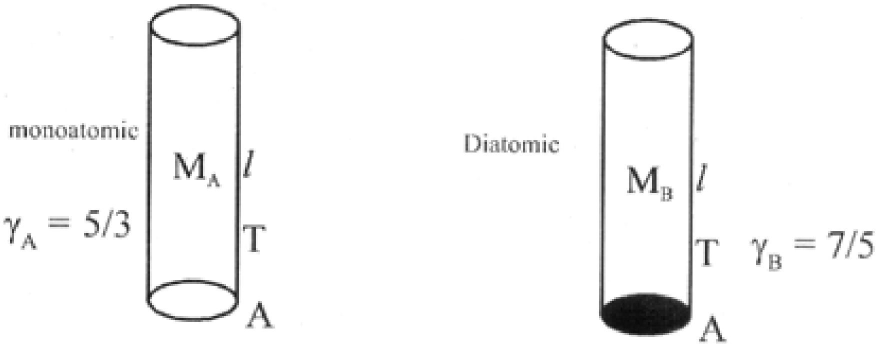
$\Rightarrow a = 1$, b is any real number.

For $a = 1$, $b \in R$, gof is continuous

$$\Rightarrow gof(x) = \begin{cases} x + 2 & , x < -a \\ x^2 + b & , -a \le x < 1 \\ (x - 2)^2 + b, & x \ge 1 \end{cases}$$

So, gof is differentiable at $x = 0$ if $a = 1$, $b \in R$.

Q.1



(a) Frequency of second harmonic in A is $n_{A_2} = \frac{v_A}{\ell} = \frac{1}{\ell}\sqrt{\frac{\gamma_A RT}{M_A}}$

Frequency of third harmonic in is B is $n_{B_3} = \frac{3v_B}{4\ell} = \frac{3}{4\ell}\sqrt{\frac{\gamma_B RT}{M_B}}$

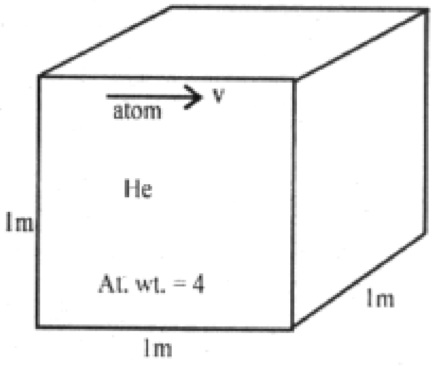
given $n_{A_2} = n_{B_3}$

$$\Rightarrow \quad \sqrt{\frac{\gamma_A}{M_A}} = \sqrt{\frac{9\gamma_B}{16M_B}} \Rightarrow \frac{M_A}{M_B} = \frac{16}{9}\frac{\gamma_A}{\gamma_B} = \frac{16 \text{ x } 5 \text{ x } 5}{9 \text{ x } 3 \text{ x } 7} = 2.116$$

(b) Now the fundamental frequency of both the pipes is $\frac{v}{2\ell}$ where v is the respective velocities thus

$$\frac{n_{A_f}}{n_{B_f}} = \frac{v_A}{v_B} = \sqrt{\frac{\gamma_A M_B}{\gamma_B M_A}} = \sqrt{\frac{\gamma_A}{\gamma_B} \text{x} \frac{9\gamma_B}{16\gamma_A}} = \frac{3}{4}$$

Q2.



$P = 100\ \text{Nt/m}^2$

given that an atom makes 500 hits /sec.

thus its rms speed is

$$v = \frac{500 \times 1 \times 2}{1} = 10^3 \text{ hits /sec.}$$

(a) temp. of gas can be given as $10^3 = \sqrt{\frac{3RT}{M}}$ or $T = \frac{4 \times 10^{-3} \times 3}{3 \times 25} = 160K$

(b) kinetic energy per atom is on an average $= \frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 160$

$$= 3.312 \times 10^{-21} \text{ Joule}$$

(c) total mass of the can be obtained as

$$PV = nRT$$

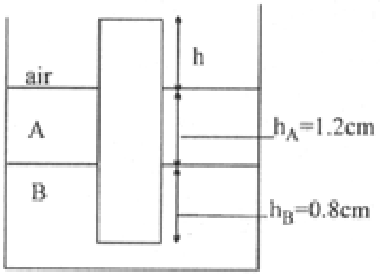
$$\text{or } 100 \times 1 = \frac{m}{4 \times 10^{-3}} = \frac{25}{3} = 160$$

$$\text{or} \quad m = \frac{4 \times 3 \times 10^{-3} \times 100}{25 \times 160} = 3 \times 10^{-4} = 0.3 \text{ gm}$$

Q3. $d_{cyl} = 0.8\ \text{gm/cm}^3$
$d_A = 0.7\ \text{gm/cm}^3$
$d_B = 1.2\ \text{gm/cm}^3$

(a) force exerted by liq. A on cylinder is $F = 0$ as no vertical part of cylinder is in contact.

(b) If S is the area of cross section of cylinder,
we have for its equilibrium



$$S\,(h + h_A + h_B)\,d_{cyl.} = h_A S\, d_A + hAS\, h_B$$

$$\text{we have } h = \frac{(h_A d_A + h_B d_B)}{d_{cyl}} - (h_A + h_B)$$

$$= \frac{1.2 \times 0.7 + 0.8 \times 1.2}{0.8} - (2) \quad = \frac{0.84 + 0.96}{0.8} - 2$$

$h = 0.25$ cm

(c) When cylinder is depressed. The height of cylinder in liquid A & B are
$h_A = 1.2$ cm & $h_B' = 1.05$ cm

$$\text{Thus net buoyancy force on cylinder is } F_{up} = \frac{(h_A d_A g + h_B' d_B g)S}{(h_A + h_B')S d_{cyl}} - g$$

$$= \frac{1.2 \times 0.7 \times 10 + 1.05 \times 1.2 \times 10}{2.25 \times 0.8} - 10 = 1.66\ \text{m/sec}^2$$

Q.4

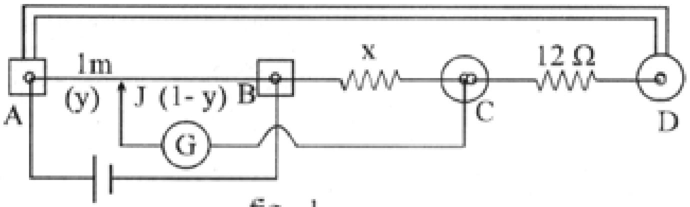


fig - 1

(a) No. As at the time of balancing the bridge, current in galvanometer is zero so we do not need a unidirectional galvanometer. In unidirectional galvanometer it is also difficiult to get null deflection point.

(b) Figure is shown above or it can also be like the

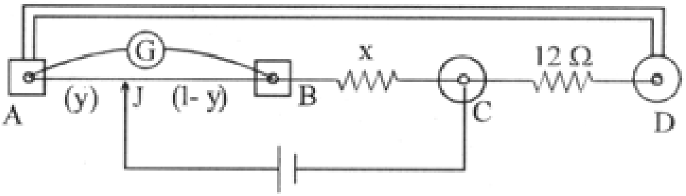


fig - 2

(c) for balanced wheat stone bridge, we must have

$$\frac{y}{1-y} = \frac{12}{x} \quad \Rightarrow \quad x = \frac{12}{y}(1-y) = \frac{12}{0.6} \times 0.4 = 8\ \Omega \quad \textbf{Ans.}$$

Q5.(a) H-like atom is emitting six radiations
let the quantum no. of –0.85 eV shells is n
then that of –0.544 eV must be n + 3
as the transistors between these two levels give six radiations, then diff. must be 3.(eg. from 4 to 1)

we have $\frac{13.6 \text{ x } z^2}{n^2} = 0.85$ and $\frac{13.6z^2}{(n+3)^2} = 0.544$

dividing we get $\frac{(n+3)^2}{n^2} = \frac{0.85}{0.544} \Rightarrow \frac{n+3}{n} = 1.25$

$$n + 3 = 1.25\,n$$

$$0.25n = 3$$

$$n = 12$$

$$\Rightarrow \quad n + 3 = 15$$

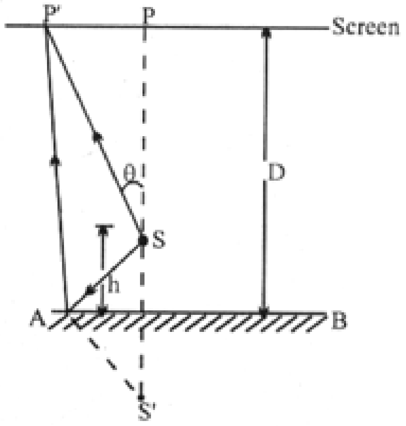
$$\Rightarrow \quad 13.6 \text{ x } z^2 = 0.85 \text{ x } (z)^2$$

$$\Rightarrow \quad z^2 = 9 \Rightarrow z = 3 \text{ ans.}$$

(b) Smaller wavelength corresponds to maximum energy transistor
⇒ n = 15 to n = 12
⇒ energy radiated is $\Delta E = (-0.544) - (-0.85) = 0.306$ eV

$$\Rightarrow \lambda = \frac{hc}{\Delta E} = \frac{12400}{0.306} = 40522.87 \text{ Å}$$

Q6.



$$\lambda = 6000\text{Å} = 600\text{nm}$$

(a) As S is a point source fringes are observed on screen due to light coming from S & its image S ′. If at P′ we discuss, what be the phase difference will remain same in a cone of half angle thus fringes will be circular.

(b) Intensity from S is if I, from S′ will be 0.36 I, thus, we have

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = \left(\frac{1.6}{0.4}\right)^2 = 16$$

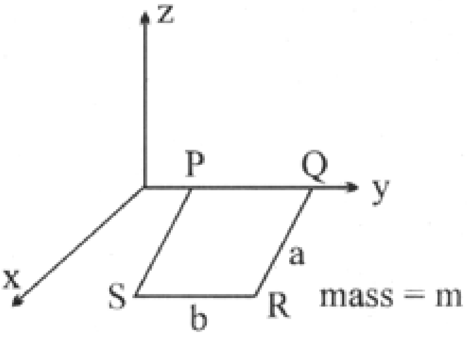
(c) If at P there is max ⇒ at P path difference is –

$$\Delta = 2h + \lambda/2 \quad = N\lambda$$

for again receiving a maxima at P, h must be increased by $\lambda/2$ so that Δ will increase by λ

⇒ disp. of AB is $\lambda/2 = 3000\text{Å}$ Ans.

Q.7 (a) $B = (3\hat{i} + 4\hat{k})B_0$



As loop is in equilibrium

$\Rightarrow \quad \Sigma \tau = 0$

or $\quad \vec{\tau}_{\text{magnetic}} + \vec{\tau}_{\text{mg}} = 0$

$$\vec{\tau}_{\text{mg}} = \left(mg \times \frac{a}{2}\right)\hat{j}$$

If magnetic moment of loop is $\vec{M} = m\hat{k}$ (as $\vec{M}$ is either in z or in – z direction)

$$\Rightarrow \quad \vec{M} \times \vec{B} = -mg \times \frac{a}{2}\hat{j}$$

$$\text{or} \quad M\hat{k} \times (3\hat{i} + 4\hat{k})B_0 = -mg\frac{a}{2}\hat{j}$$

$$3M B_0 \hat{j} = -mg\frac{a}{2}\hat{j} \qquad \text{....(1)}$$

Thus we must have magnitude has M as – ve or $\vec{M}$ is in – z direction. Thus current in loop PQRS is clockwise from P to QRS.

(b) Magnetic force on arm RS is $\vec{F} = I\left(\vec{b} \times \vec{B}\right)$

$$\vec{F} = I\left[(-b\hat{j}) \times (3\hat{i} + 4\hat{k})B_0\right] \qquad (\text{as } \vec{b} \text{ is} - b\hat{j})$$

$$\vec{F} = BI_0\left[3b\hat{k} - 4b\hat{i}\right]$$

$$\vec{F} = BI_0 b\ (3\hat{i} - 4\hat{k})$$

$$|\vec{F}| = 5\,BI_0 b$$ **Ans.**

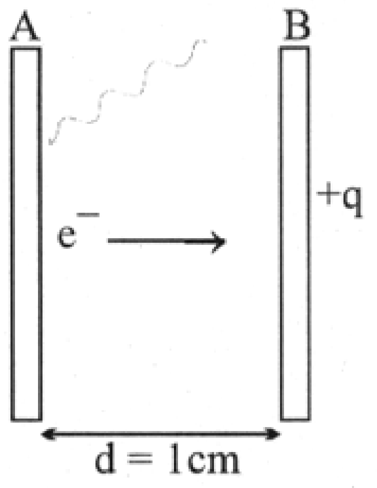
(c) From equation (1) we have

$$3MB_0 = mg\,\frac{a}{2} \qquad \text{in magnitude}$$

$$3\,I\,b\,B_0 = \frac{mg}{2}$$

$$I = \frac{mg}{6bB_0}$$ **Ans.**

Q8.



Area $A = 5 \times 10^{-4}\ m^2$

$q = 33.7 \times 10^{-12}$ C

& $\frac{hc}{\lambda} = 5eV$

given 10^{16} photon falls /sec/ m^2

photo efficiency is $\eta = 1$ out of 10^6 ph

work function of A $\phi = 2eV$

(a) No. of photo electrons upto t = 10 sec.

$$N = \frac{10^{16}}{10^6} \times 5 \times 10^{-4} \times 10 = 5 \times 10^7\ e^-$$

(b) Charge emitted in 10 sec. is $q = 5 \times 10^7 \times 1.6 \times 10^{-19} = 8 \times 10^{-12}$ coul

now charge on plate A is $q_A = +8 \times 10^{-12}$ coul

on plate B is $q_B = +(337 - 8) \times 10^{-12} = 25.7 \times 10^{-12}$ coul

EF between the two plates now is

$$E = \frac{q_B - q_A}{2A\,\epsilon_0} = \frac{17.7 \times 10^{-12}}{2 \times 5 \times 10^{-4}} = 2000\ V/m$$

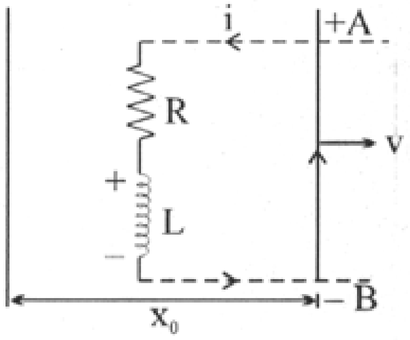
(c) maximum KE of the photo electron just emitted by plate A is $KE_{max} = 5 - 2 = 3eV$

potential difference at this instant between plates A & B is = E.d = 2000 x 1 cm

= 20 volts

⇒ KE of e^- when reaches B is $KE_{at\ B} = 3eV + 20eV = 23\ eV$

Q.9 (a) Let i be the current in circuit when wire AB is sliding at v towards right, emf induced in wire is when it is at dustance x from I_0



$$e_{AB} = \frac{\mu_0 I_0}{2\pi x}.l.v = \frac{d\phi}{dt}$$ (if v is the velocity of AB at this instant)

for circuit loop we can write

$$e_{AB} - i\,R - L\,\frac{di}{dt} = 0$$

$$\frac{d\phi}{dt} - i\,R - L\frac{di}{dt} = 0$$

(b) Charge flown through the resistance is $q = \int_0^T i\,dt$

or for circuit loop we can write total charge flown is

$$\Delta q = \frac{\Delta\phi}{R} = \frac{(\phi_1 - Li_1) - \phi_2}{R}$$

where ϕ_1 = final flux through circuit when $x = 2x_0$
& ϕ_2 = initial flux through circuit when $x = x_0$

$$\Rightarrow \quad \phi_1 - \phi_2 = \frac{\mu_0 I_0 l}{2\pi}\,ln\,(2)$$

$$\Rightarrow \quad \Delta q = \frac{\frac{\mu_0 I_0 l\,ln(2)}{2\pi} - Li_1}{R}$$

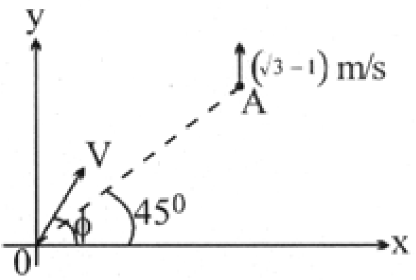
(c) Given that at time t rod is stopped, now transient decays & decaying current is

$i = i_0 e^{-Rt/L}$ where $i_0 = i_1$ & $i = i_1/4$ at $t = 2T$

$$\Rightarrow \quad i_1/4 = i_1 e^{-R(2T)/L}$$

$$2\,ln\,2 = \frac{2RT}{L} \quad \Rightarrow \quad \frac{L}{R} = \frac{T}{ln\,(2)}$$

Q10.



(a) As ball will hit A, it appears to A that ball is moving towards A from O. Thus the apparent angle will be 45^0

(b) If velocity of ball relative to surface is v. Velocity w.r.t. A is

$$v_x = v\cos\phi$$

$$v_y = v\sin\phi - \left(\sqrt{3}-1\right)$$

we have $\theta = 45^0 = \tan^{-1}\dfrac{v_x}{v_y} = \tan^{-1}\left(\dfrac{v\cos\phi}{v\sin\phi - \left(\sqrt{3}-1\right)}\right)$

given that $\phi = \dfrac{4\theta}{3} = 60^0$

$$\Rightarrow \tan 45^0 = 1 = \frac{v\cos\phi}{v\sin\phi - \left(\sqrt{3}-1\right)}$$

$$v\sin(60^0) - \left(\sqrt{3}-1\right) = v\cos 60^0$$

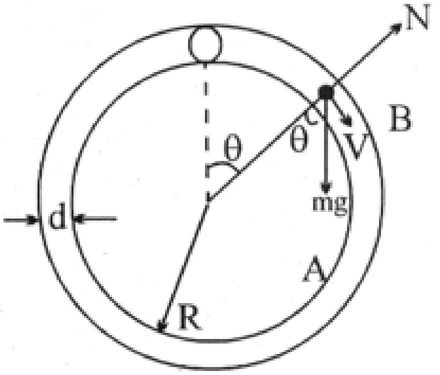
$$\frac{v\sqrt{3}}{2} - \sqrt{3} + 1 = \frac{v}{2}$$

$$v\sqrt{3} - \sqrt{3} \text{ x } 2 + 2 = v$$

$$v\left(\sqrt{3}-1\right) = 2\left(\sqrt{3}-1\right)$$

v = 2m/sec.

Q11.



(a) Let when ball is at an angle θ, its velocity is given by

$$V = \sqrt{2gR(1-\cos\theta)}$$

for its radial equilibrium we have

$$mg\cos\theta = N + \frac{mv^2}{R} \Rightarrow N = mg\cos\theta - \frac{m}{R}\,(2gR(1\text{-}\cos\theta))$$

$N = 3mg\cos\theta - 2mg$

(b) We know that N_A wll be zero when

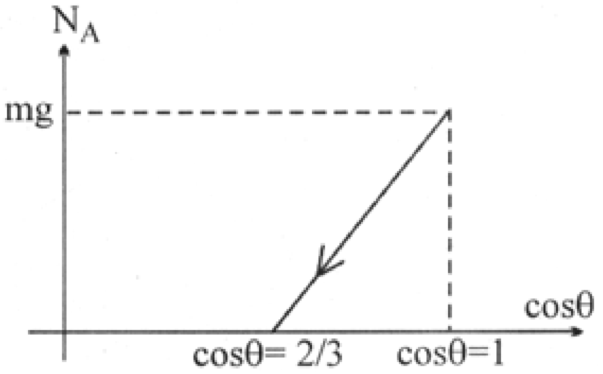
$$\frac{mv^2}{R} = mg\cos\theta \Rightarrow \cos\theta = 2/3$$

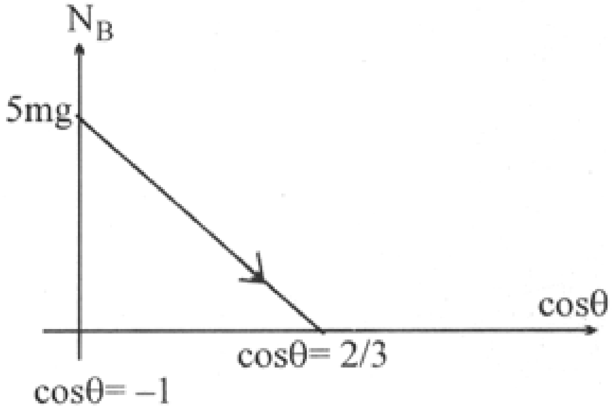
now N_B appears as we will have after this instant

$$N_B + mg\cos\theta \Rightarrow \frac{mv^2}{R} = 2mg - 2mg\cos\theta$$

or $N_B = 2mg - 3mg\cos\theta$

graphs are as follows –

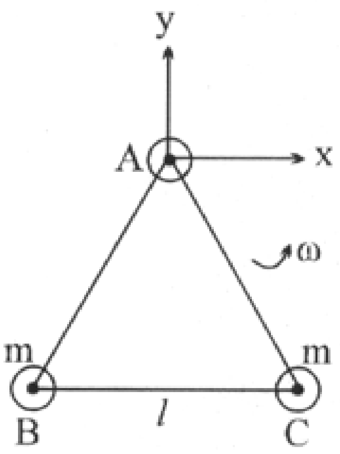




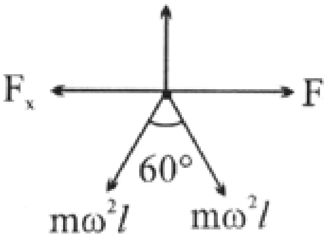
Q.12

(a) Force exerted by hinge in horizontal direction must be equal to the centripetal force required for circular motions of particle B & C as

$$F_H = 2\ m\ \omega^2 l \cos 30^\circ = \sqrt{3}\, m\ \omega^2 l$$



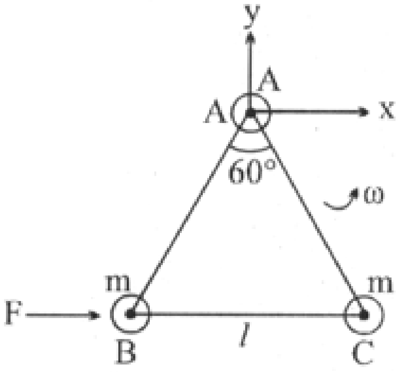
(b) Now let hinge exerts F_x & F_y force on body. We have



Let body rotates with angular a_{cm}, we have

$$F \times \frac{\sqrt{3}l}{2} = 2mt^2.\alpha$$

or $$\alpha = \frac{\sqrt{3}F}{2ml}$$



linear acceleration of cm of body is

$$a_{cm} = \alpha \times \frac{l}{\sqrt{3}} = \frac{\sqrt{3}F}{4ml} \times \frac{l}{\sqrt{3}} = \frac{F}{4M}$$ in horizontal direction from above F & D we have

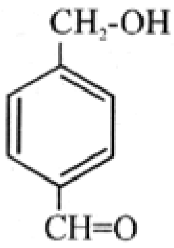
$$F - F_x = 3\ m\left(\frac{F}{4M}\right) = \frac{3F}{4}$$

$$F_x = F - \frac{3F}{4} = \frac{F}{4}$$ **Ans.**

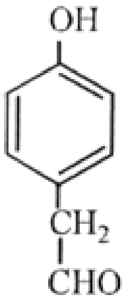
In y direction there is no acceleration of body at this instant thus

$$F_y = 2\ m\ \omega^2 l \cos 30^\circ = \sqrt{3}\ m\ \omega^2 l.$$

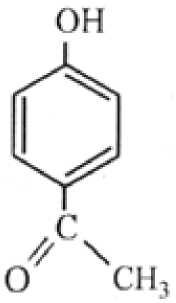
Q1. (A)



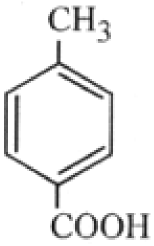
(B)



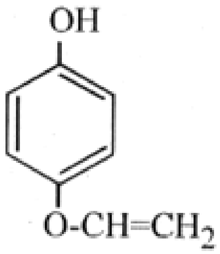
(C)



(D)



(E)



Q2.(i) 500ml of 0.2M CH_3COOH

500 ml of 0.2M HCl

V = 500 + 500 = 1000ml, M(CH_3COOH) = 0.1 M; M(HCl) = 0.1M

$$\underset{0.1\,(1-\alpha)}{CH_3COOH} \Leftrightarrow \underset{0.1\,\alpha}{CH_3COO^-} + \underset{0.1\,\alpha}{H^+}$$

$$HCl \longrightarrow H^+ + Cl^-$$

0.1	0	0
0	0.1	0.1

$$\underset{0.1\,(1-\alpha')}{CH_3COOH} \Leftrightarrow \underset{0.1\,\alpha'}{CH_3COO^-} + \underset{(0.1\,\alpha' + 0.1)}{H^+}$$

$$k_a = \frac{[CH_3COO^-]\,[H^+]}{[CH_3COOH]} = \frac{(0.1\,\alpha' + 0.1)\,0.1\,\alpha'}{0.1(1-\alpha')}$$

$$k_a = 1.75 \times 10^{-5}$$

$$1.75 \times 10^{-5} = \frac{0.1\,\alpha'(0.1\,\alpha'+0.1)}{0.1(1-\alpha')}$$

$$\alpha' = 1.75 \times 10^{-4} \text{ (approx)}$$

(ii) mol of NaOH = 6/40 = 0.15

molarity of NaOH = 0.15M

$HCl + NaOH \longrightarrow NaCl + H_2O$

HCl	NaOH	NaCl	H_2O
0.1	0.15	0	0
0	0.05	0.1	0.1

$CH_3COOH + NaOH \longrightarrow CH_3COONa + H_2O$

CH_3COOH	NaOH	CH_3COONa	H_2O
0.1	0.05	0	0.1
0.05	0	0.05	0.15

pH = pKa + log salt/Acid

$pH = -\log(1.75 \times 10^{-5}) + \log(0.05 / 0.05)$

pH = 4.76

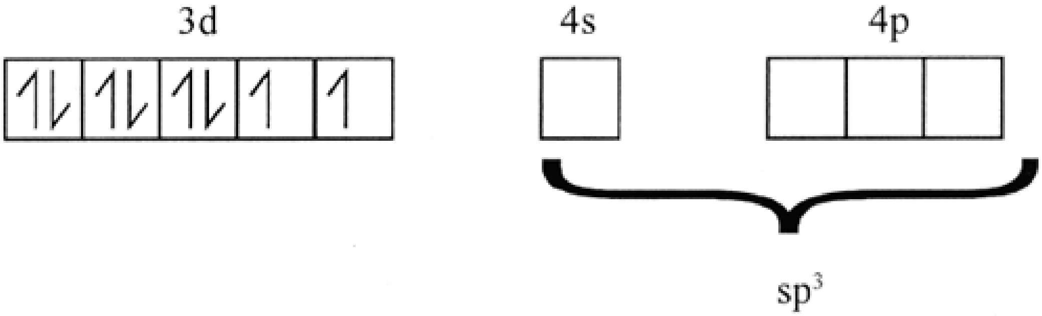
Q3. $[NiCl_4]^{2-}$

$Ni \rightarrow 28 \rightarrow 3d^8, 4s^2$

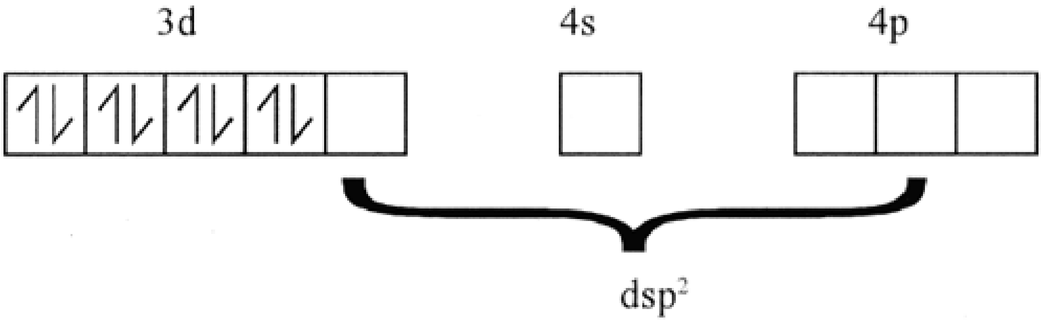
$Ni^{2+} \rightarrow 3d^8$

Cl^- is high spin ligand and CN^- is low spin

$[NiCl_4]^{2-}$



$[Ni(CN)_4]^{2-}$



$[NiCl_4]^{2-}$ is tetrahedral

$[Ni(CN)_4]^2$ is square planer

$\mu = \sqrt{n(n+2)}$ BM where n is no. of unpaired electron

$\mu\ [NiCl_4]^{2-} = \sqrt{2(4)} = \sqrt{8} = 2\sqrt{2}$ BM

$\mu\ [Ni(CN)_4]^2 = \sqrt{0(0+2)} = 0\ BM$

Q4.(a) $\frac{r_{vapour}}{r_{O_2}} = \sqrt{\frac{32}{M_{vapour}}} = 1.33$

$$M_{vapour} = \frac{32}{(1.33)^2} = 18.09\ gm/mol$$

$$V = \frac{Mass}{Density} = \left(\frac{18.09 \times 10^{-3}}{0.36}\right) m^3 = 50.25\ litre$$

$$PV = ZnRT$$

$$P = \frac{Z \cdot wt}{V \cdot M} \times RT$$

$$P \times M = Z\rho RT$$

$$Z = \frac{18.09 \times 1}{0.36 \times 0.0821 \times 500} = 1.22$$

Z > 1 so repulsive force will dominate.

(b) $KE = \frac{3}{2} kT$

$$= \frac{3}{2} \times 1.38 \times 10^{-23} \times 1000$$

$$= 2.07 \times 10^{-20}\ \text{Joules.}$$

Q5. (i) $Al_4C_3 + 6H_2O \rightarrow 4Al(OH)_3 + 3CH_4$

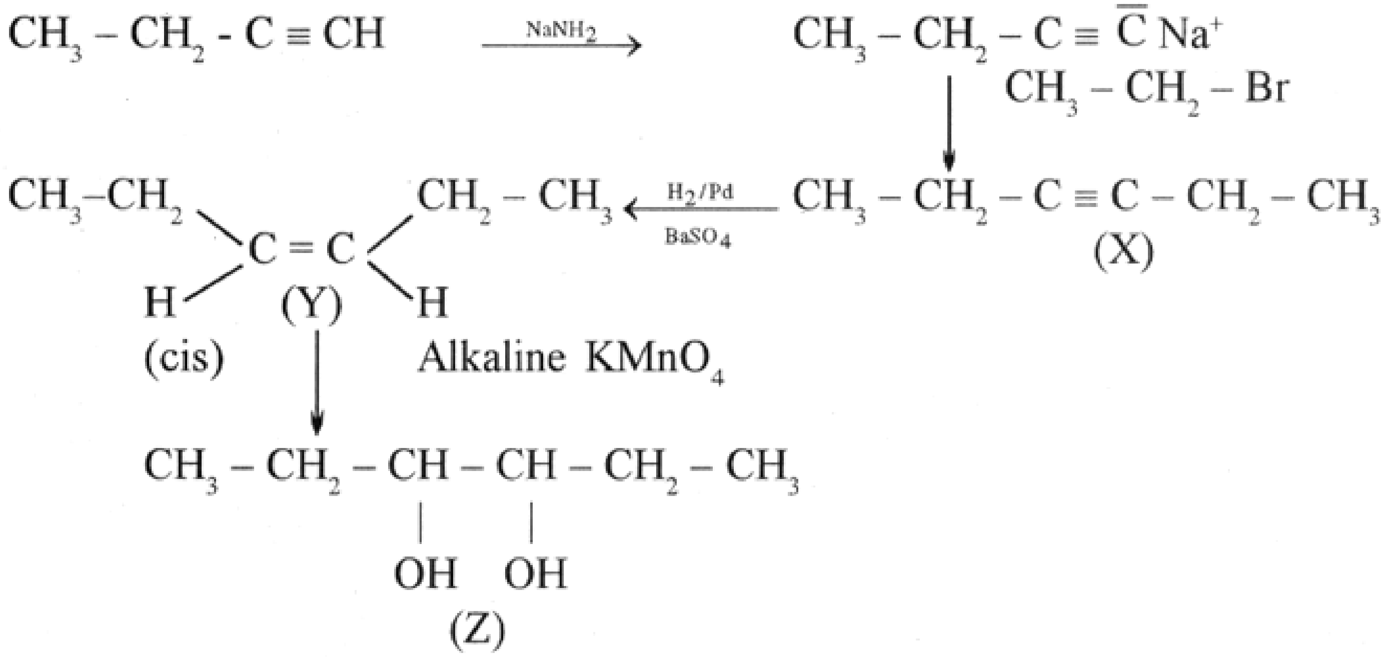
(ii) $CaCN_2 + 5H_2O \rightarrow CaCO_3 + 2NH_4OH$

(iii) $BF_3 + 3H_2O \rightarrow B(OH)_3 + 3HF$

(iv) $NCl_3 + 4H_2O \rightarrow NH_4OH + 3HOCl$

(v) $3XeF_4 + 6H_2O \rightarrow 2Xe + XeO_3 + 12\ HF + \frac{3}{2}\ O_2$

Q6.



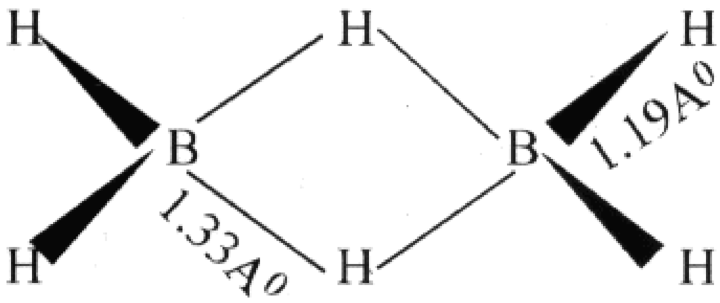
Compound Z consist of plane of symmetry. So it is optically inactive.(Meso)

Q7.

$$\underset{\text{Borax}}{Na_2B_4O_7 \cdot 10H_2O} + 2HCl \longrightarrow 2NaCl + 4H_3BO_3 + 5H_2O$$

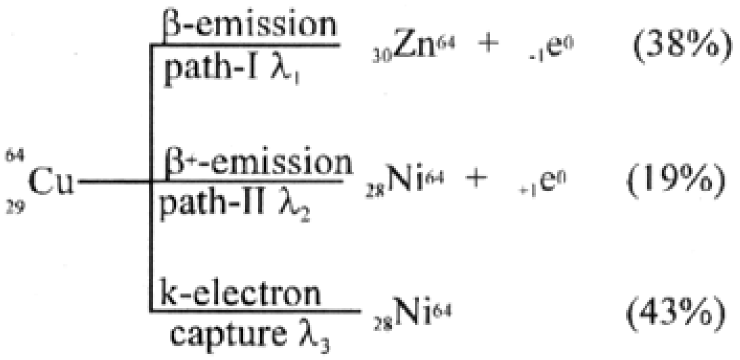
$$\underset{\text{o-Boric acid}}{H_3BO_3} \xrightarrow[\text{Red Heat}]{\Delta} \underset{\text{Boric anhydride}}{B_2O_3} + H_2O$$

$$B_2O_3 + Mg \longrightarrow \underset{\text{Boron}}{B} + MgO$$



$$B_2H_6 + HCl \longrightarrow B_2H_5Cl + H_2$$

Q8.



$$ {}_{29}Cu^{64} \xrightarrow[\text{path-I } \lambda_1]{\beta\text{-emission}} {}_{30}Zn^{64} + {}_{-1}e^{0} \quad (38\%) $$

$$ {}_{29}Cu^{64} \xrightarrow[\text{path-II } \lambda_2]{\beta^{+}\text{-emission}} {}_{28}Ni^{64} + {}_{+1}e^{0} \quad (19\%) $$

$$ {}_{29}Cu^{64} \xrightarrow[\text{capture } \lambda_3]{\text{k-electron}} {}_{28}Ni^{64} \quad (43\%) $$

$$ n + {}_{-1}e^{0} + \overline{\nu} \rightarrow \pi $$

$$ \lambda_{net} = \lambda_1 + \lambda_2 + \lambda_3 $$

$$ \text{Fractional yield for path (I)} = \frac{\lambda_1}{\lambda_{net}} \Rightarrow \frac{\dfrac{\cancel{0.6931}}{(t_{1/2})_1}}{\dfrac{\cancel{0.6931}}{(t_{1/2})_{net}}} = \frac{(t_{1/2})_{net}}{(t_{1/2})_1} $$

$$ \text{Half life for Path I} = \frac{(t_{1/2})_{net}}{\text{fractional yield}} = \frac{12.8}{0.38} = 33.684 \text{ hr} $$

$$ \text{Half life for path II} = \frac{12.8}{0.19} = 67.368 \text{ hr} $$

$$ \text{Half life for path III} = \frac{12.8}{0.43} = 29.767 \text{ hr} $$

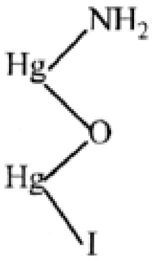
Q9. (X) NH_4Cl (A) CrO_2Cl_2 (B) Na_2CrO_4 (C) $PbCrO_4$

(D) $HgOIHgNH_2$

$$NH_4Cl + K_2Cr_2O_7 + 3H_2SO_4 \longrightarrow K_2SO_4 + (NH_4)_2SO_4 + 2CrO_2Cl_2\uparrow + 3H_2O$$

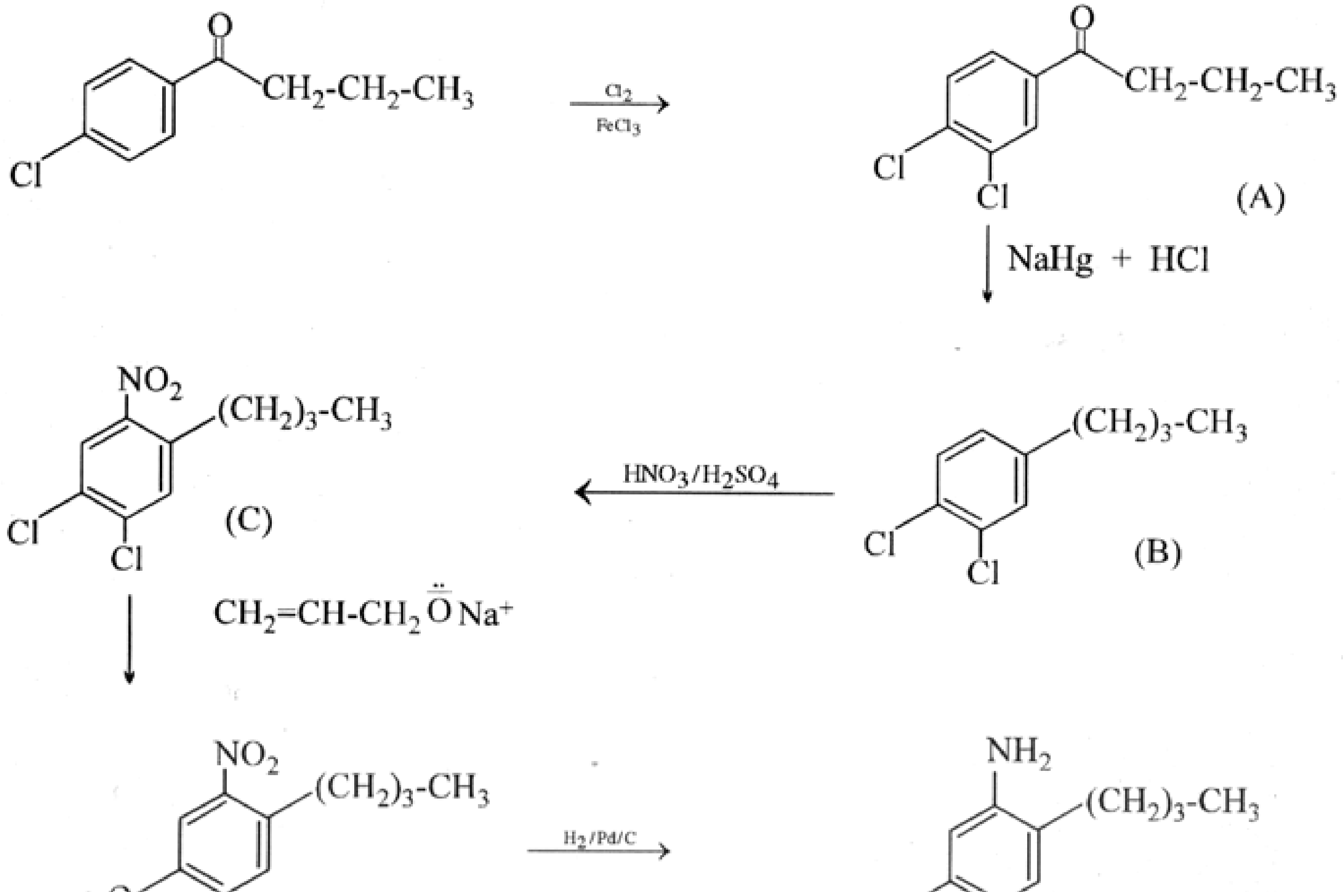
$$CrO_2Cl_2 + 4NaOH \longrightarrow 2NaCl + Na_2CrO_4 + 2H_2O$$

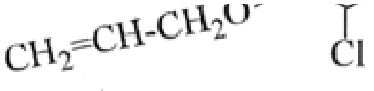
$$Na_2CrO_4 + (CH_3COO)_2Pb \longrightarrow \underset{\text{yellow}}{PbCrO_4\downarrow} + CH_3COONa$$

$\underset{(X)}{NH_4Cl} + 2K_2HgI_4 + KOH \longrightarrow$  $+ 7KI + KCl + H_2O$

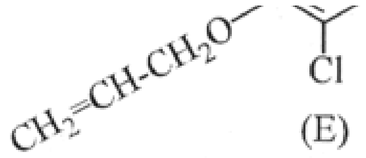
Brown ppt.

Q10.





(D)



(E)

Q11. $CH_3{-}CH_2{-}CH_2{-}CH = CH - CH = CH - (CH_2)_8 - CH_2 - OH$

(Bombykol)

In Bombykol total no. of Double bond is 2

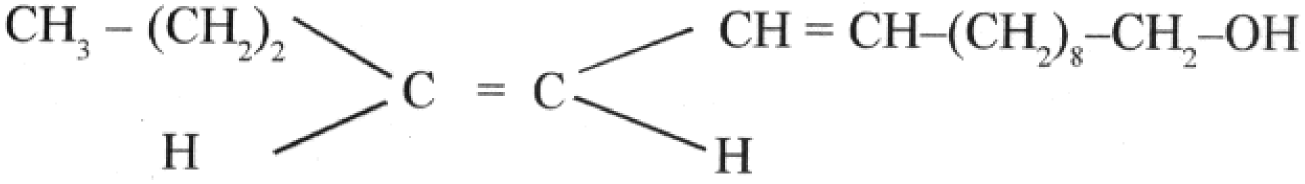
$C_{16}H_{30}O$

$\beta = (16 + 1) - 30/2$

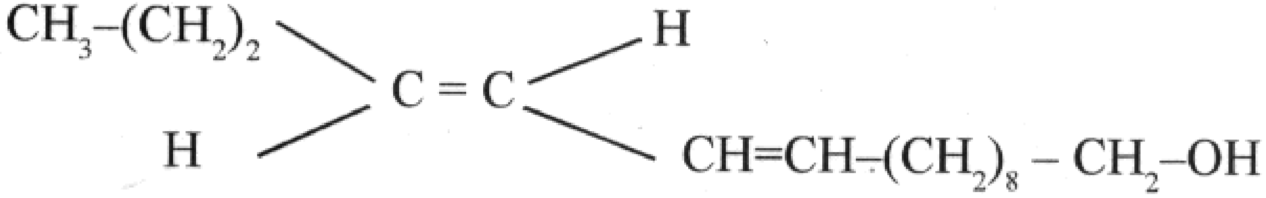
$\beta = 2$ (Degree of unsaturation)

(A) $CH_3 - (CH_2)_{14} - CH_2 - OH$

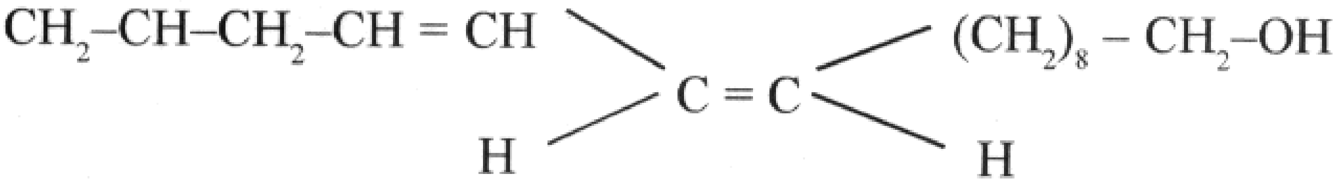
Total no. of Geometrical Isomer in Bombykol is 4



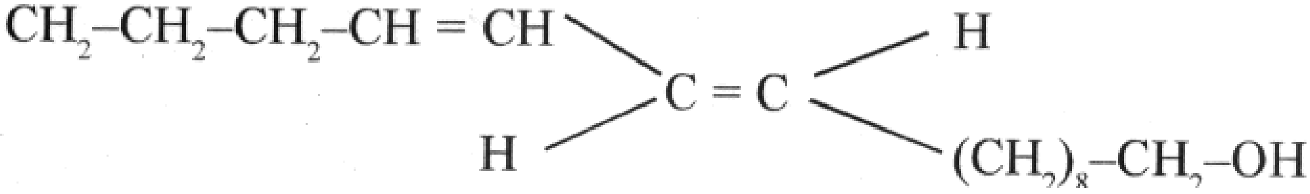
(Cis)



trans



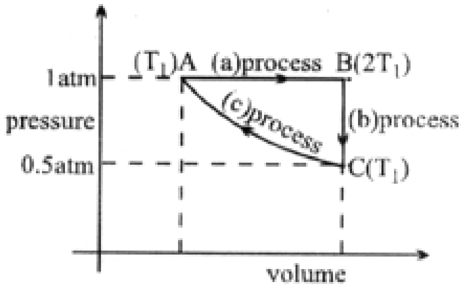
cis



trans

Q12.

(i)



where T_1 & T_2 are temperature when gas is at stage A & B.

(ii) Total work (w) is the area under the curve

$$= 1 \text{ atm } (40 - 20)\text{L} - 2.303 \times 2 \times 0.0821 \times 27 \log\frac{20}{40}$$

$$= 51.076 \text{ lit atm}$$

$$= 5.1723 \times 10^3 \text{ Joule}$$

$\Delta Q = \Delta U + \Delta w$ (First law of thermodynamics)

$\Delta U = 0$ (cyclic process)

$\Delta Q = w = 5.1723 \times 10^3$ Joule

(iii) $\Delta U = 0$

$$\Delta H_{net} = f \times \frac{n}{2} R\Delta T + w_{A\to B} + f\,\frac{n}{2}\,R\,\Delta T + w_{B\to C} + 0 + 0$$

$$= 20 \times 1 \text{ L atm}$$

$$= 2.025 \times 10^3 \text{ Joule}$$

$\Delta S = 0$ for cyclic process